Some ways to get a piece of Pi Day action

Alice Richardson
University of Canberra
Alice.Richardson@canberra.edu.au

Judith Ascione
University of Canberra
Judith.Ascione@canberra.edu.au

Valerie Barker
University of Canberra
Valerie.Barker@canberra.edu.au

Introduction

22 July ▸ 22–7 ▸ 22/7 ▸ 3 1/7 ▸ 3.14159... ▸ π

In many parts of the world Pi Day (www.piday.org) is celebrated on March 14 (3.14), but because of the day-month ordering of dates in Australia, and because March is very close to the start of the academic year, Australians prefer to celebrate Pi (Approximation) Day on 22 July (22/7).

Thirty-eight Year 8 students (aged 13–14 years) from two local high schools joined ESTeM Maths and Education staff on Tuesday 22nd July 2014 to celebrate Pi Day. Each group of students made their way to Building 6 following carefully devised π-trails across the campus, and on arrival enjoyed a morning tea of pi-kelets. For three hours they participated in a range of activities to celebrate this remarkable number, followed by a lunch of pi-zzas and pi-eces of fruit, especially pi-neapples.

Academic context

In most Australian states in the last two decades, school students have met the concept of π in Year 8. The Australian Curriculum Mathematics (ACARA, 2014) has retained this convention, as shown in Figure 1.

Measurement and Geometry (Year 8)

Using units of measurement

ACMMG197 Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Elaborations

- Investigating the circumference and area of circles with materials or by measuring, to establish an understanding of formula.
- Investigating the area of circles using a square grid or by rearranging a circle divided into sectors

Number and Algebra (Year 8)

Real numbers

ACMNA186 Investigate the concept of irrational numbers, including π.

Elaborations

- Understanding that the real number system includes irrational numbers.

Figure 1. The Australian Curriculum Mathematics syllabus material (ACARA, 2014)

The Year 8 achievement standard (ACARA, 2014) includes the following: "They [students] describe rational and irrational numbers …They [students] name the features of circles and calculate the areas and circumferences of circles".

Many students come into Year 8 with at least a passing knowledge that there is a number known as π, that it has an infinite number of decimal places (the decimal places "go on forever", and that it has something to do with circles. A number of students will 'know' (that is, be able to recite) the formula for the circumference and area of a circle and a number of the digits for π.

A constructivist pedagogical perspective, drawing on the work of theorists such as Piaget, Vygotsky and Bandura (Killen, 2013, pp. 42–45), deems it essential to provide a learning environment and activities so that students are constructing their knowledge and understandings rather than just being given them. In this context, rather than simply giving students a definition of π, the provision is made of opportunities for students to construct an understanding of π as a ratio. Given the prolific use of calculators which present π as an apparently terminating decimal (rather than as a rational number approximation), the notion of π as an irrational number is probably not emphasised nor even paid attention to in many classrooms.

Activities

There are plenty of online resources listing potential Pi Day activities (some suggested examples are www.educationworld.com, www.nctm.org and teachpi.org). For our purposes, we selected six activities that met our combined criteria of (a) feasibility, (b) pedagogical soundness, and (c) pedagogical appropriateness for the Australian school system.

We actively chose not to consider developing (in any other way) a formula for the area of the circle, as much as anything as there would have been no time. Moreover, there had been no opportunity to allow consolidation of the concept of π relationship to have been made before moving to applications of that relationship for the calculation of both circumference and area of a circle. Our choice, as per the theme of the day, was to consider the nature of π itself.

We considered a number of activities. Those that we excluded had common grounds for being eliminated: all involved longer periods of time to be carried out successfully than was available; knowledge of sophisticated algebraic processes was assumed; higher order statistical and probability understandings were assumed and/or necessary; and higher levels of literacy than we felt confident would be evident in all but a small percentage of the group. Interestingly, there was little if any connection made between the rational number ($\frac{22}{7}$) approximation for π and the 3.14… approximation, which is by far the most widely used form today given the use of calculators. This was alluded to only in the History of Pi rotational activity.

There are always a number of activities available for this sort of day; it is not necessarily easy to select those that typically suit or which can be readily adapted to a mixed ability cohort such as this, especially where we did not know the students. Time constraints were another significant factor in our decisions.

Measuring the circumference of the Earth (CircumEarth)

The headline activity for the day was measuring the circumference of the Earth. Students were guided by presenters who employed Eratosthenes' method of measuring the length of a shadow at midday at two locations on the same longitude.

Eratosthenes measured the circumference of the earth by measuring the length of the shadows at noon on the summer solstice. Russell (2007) provides details for calculating the Earth's circumference using Eratosthenes' method in his website, which is listed as part of the references in this article. Eratosthenes chose one place to be on the Tropic of Cancer because he knew that there was no shadow at noon on that day. This allowed him

to calculate the angle between the two places, measured from the centre of the Earth. Eratosthenes had specially trained surveyors to measure the distance between the two places and this allowed him to calculate the circumference of the Earth. Having two places on the same longitude makes the calculations more straightforward but is not strictly necessary.

To replicate measurements for this activity was a logistically complex process and took several weeks to coordinate, along with fervent wishes for fine weather on the day in two locations 1000 km apart. In Canberra, ACT we teamed up with a high school in Mackay, Queensland. A Skype connection was used to convey the data from Queensland, and to allow the calculation to take place in Canberra. Through the implementation and use of communication technology it was not necessary to wait for the summer solstice, and the comparison of results could occur immediately. Figure 2 shows students engaging with Pi Day presenters to determine the length of a shadow cast by a 1 metre ruler in preparation for measuring the circumference of the Earth.

Figure 2. Students and Pi Day presenters measure the length of a shadow cast by a 1 metre ruler in preparation for measuring the circumference of the Earth.

Whole group activity (round objects)

Constructing an understanding of π as the ratio of the circumference to the diameter of a circle is a common classroom activity—in use by one of the authors (Barker) in this context since the 1980s at least; indeed, such activities are crucial if teachers are to avoid a mindless giving of facts and formulae to our students without there being any depth of thinking and understanding.

Students worked in groups with a variety of household objects which have a circular cross-section and using a variety of instruments and procedures recorded the diameter and circumference of these. The results were tabulated, and for each pair of dimensions, students applied the four operations (+, −, ×, ÷), usually in that order. Often students simply applied the operation of division (because of time constraints) but having them use all four operations reinforced that nature of the investigation which hinges on the development of a pattern in the results which appears only for the operation of division. Understandings of error of measurement, of rounding appropriately, and of checking the sense of results were also evident in the process. It was useful to have students record their group's results into one whole class collaborative set of results (on a whiteboard or similar), as this enabled any patterns to be seen more readily. Students of all ability levels were quick to discern the similarity in the responses to C ÷ d compared with the other three operations. (The understanding that this is an irrational number was less secure, and would generally need some more explicit teaching in relation to sets of numbers and their properties). There was a need for significant amounts of guided discussion at this stage, to establish the relationship of C = πd, and what

this 'means' (such as the distance around the circumference is always a little over three ('three and a bit') times the diameter. The alternative form $C = 2\pi r$ hinges on confidence in the relationship $d = 2r$, as well as some conventions of notation i.e. $C = 2\ \pi r$ rather than $C = \pi 2r$.

This activity was considered a crucial part of the Pi Day activities as it is easily adapted to cater for a wide range of learning abilities, and involves students working collaboratively in groups. Students choose the particular three or four items that they wished to measure; frequently, even those who can recite π to a number of decimal places have not made this concrete connection before.

The rotation activities

The group of 38 students were then divided into three groups which rotated through the next three half-hour activities. By the end of the day, all students had participated in and experienced every activity scheduled.

History

A 'History of π' activity sheet adapted from Binns and Carrozza (2000), and a take-home page of 'Fun Facts' (collected from various printed and electronic sources and available on request from the authors) were designed to capture the interest of the more able students. In particular, this activity assumed an understanding of the nature of the fraction as an exact number (and an understanding that a fraction is a way of representing the operation of division) compared with the limitations of equivalent decimal representations. It required facility with the standard classroom scientific calculator, and confidence in the process of converting a fraction to a decimal.

Pi bracelets

The pi bracelet activity involved threading beads into a bracelet where ten different colours of beads indicated the digits of π, in order. The point of threading the beads in order of the digits in the decimal expansion was to highlight to students the irrational nature of π. This activity was chosen as we had been a little surprised in a previous year by how many of the boys had enjoyed making the bracelets, and we were keen to have something that would appeal to both the girls and the boys. It was also something of a surprise that most students did not seem to feel that this was an activity that was too 'childish' for them. A very practical activity also provided a counterbalance to the more intellectual engagement of several other activities. Having a tangible item, a souvenir, to take away from the day was also a factor in the decision to make the bracelets.

Demonstration

In this activity, a Professor of Science Communication demonstrated how the value of π can be approximated by the area of polygons with ever-increasing numbers of sides. This activity also tended to engage the more able students who were asked to make a series of logical connections and to generalise from the particular to the general. This activity proceeded from an understanding of π, and its inherent relationship with the circumference and the diameter thence to the development of a formula for the area of a circle.

Pi trail

This was a very engaging activity which physically and intellectually drew the students into the sphere of the day's activities. Students walked from their schools to the venue, guided by coloured π symbols where there were as many of each colour as the digit value represented. That is, there were three of the first colour, one of the second colour, four of the third colour, and so on. As with the pi bracelets, the point of following a visual representation of the digits of π reminds students that the decimal expansion of π does not repeat i.e. π is an irrational number. It relied on the assumption that a number of students would come to the day with

an awareness of π as being related to 3.14... (although not necessarily understanding that decimal approximation as an irrational number).

Evaluation

In this section, we report on the results of data collected from both the students and the pre-service teacher education students on the activities undertaken. Everyone was asked whether each activity was fun, and whether it helped them learn something about aspects of π. Answers were collected on a scale of Strongly Disagree (1) to Strongly Agree (5). Everyone was also asked what they thought was the best aspect of the day, and how the day might be improved.

Figures 3 and 4 show the number of responses of students to the statements "the [insert name of activity] was fun" and "the [insert name of activity] helped me learn something about π". The length of the red bars, for Agree and Strongly Agree, indicate that the most popular activities were the Pi Trail and the Round Objects. The least popular activities to engage with were the Demonstration and the Pi Bracelets. Activities that assisted students to learn the most were the Round Objects and the History. Pi Trail and the Pi Bracelets were evaluated by students to be activities which assisted them the least to learn more about this area of mathematics.

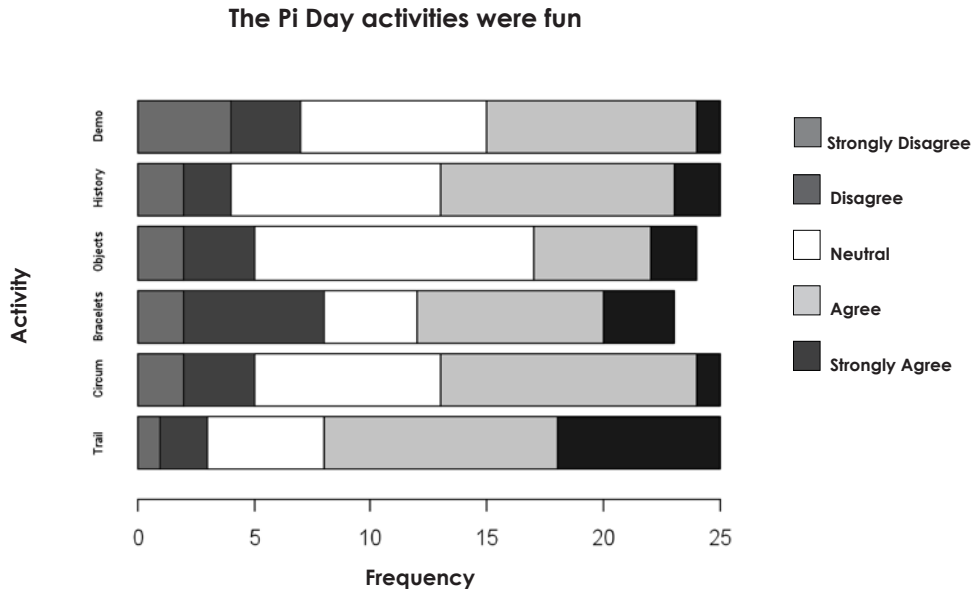

Figure 3. Bar chart of responses to the statement "The Pi Day activities were fun".

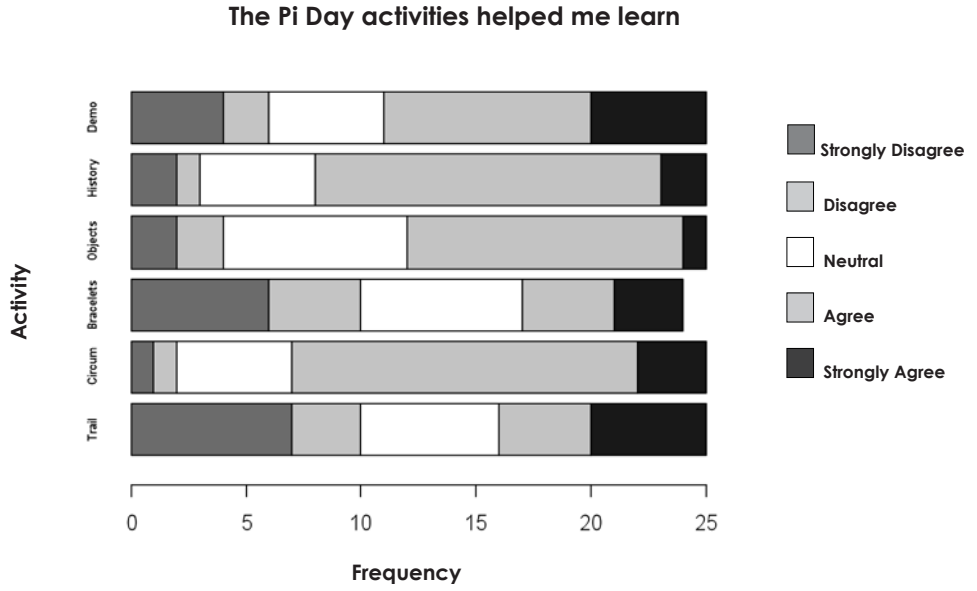

Figure 4. Bar chart of responses to the statement "The Pi Day activities helped me learn something about π".

No activity rated less than a median of 3 on either scale, showing that on average students found something to take away from all activities that they engaged in and with. There was no activity that scored above the median on both fun and helpful, showing that, unfortunately, we still have not located the "magic bullet" activity—one that is perceived as both fun and helping students to learn something.

Reflections on the Pi day activities

Measuring the circumference of the earth does not explicitly involve π, although the circumference clearly leads to the concept of π very quickly. This activity was originally incorporated into the Pi Day event in 2013, during the year of Mathematics of Planet Earth (www.mpe2013.org). It was so successful that we have kept it on.

Next time we would allow more time for the Round Objects activity, so that the discussion need not be curtailed as it was in 2014. Others wishing to arrange a Pi Day could also consider facilitating the follow-up of unfinished activities by the teachers on a later day in a classroom setting. A dynamic geometry package such as Graphmatica (www.graphmatica.com) could also be used to facilitate the discussion in the Demonstration.

The Pi Trail was the surprise package. It was a completely non-mathematical activity but it seemed to set a cheerful tone for the session, and put students into a good mood for the activities to come. One student commented that for her, "the best part of the day was definitely the treasure hunt-like arrival as it was fun and involved us actually finding our way to the pi-day part of the University." For another student, his favourite activity was "…the sprint for the next pi signs".

Food also helped to create a great atmosphere for Pi Day (Figure 5). Isn't it generally agreed that food is one of the most memorable aspects of any conference? One student said that the best aspect of the day was "…the pi-food", and another said "The pizza was the best part of the day". "Actual pies" was one suggestion for improvement.

Figure 5. Pi-kelets.

Even more activity could have been included: as one student put it, "…where we are actually doing something instead of just sitting down and doing worksheets like we do in class".

Sometimes you just cannot please everyone all of the time. There were two comments advising dropping the Pi bracelets, and one who thought it was the best part of the day.

The demonstration was regarded by one pre-service teacher as "…too difficult for many students and they struggled to maintain interest. Something more interactive could have been useful."

Conclusion

Pi Day in 2014 was celebrated in Canberra with six different activities. Student evaluation showed that on average they all achieved one of our two aims of being fun or helping to teach students to learn something related to π. We encourage others to get on board with Pi Day, whether in March or July, because there is sufficient material and plenty to learn about π.

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